PROBABILISTIC ANALYSIS OF GEOSYNTHETIC-REINFORCED UNPAVED ROAD SUBJECT TO SUBGRADE VARIABILITY

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ABSTRACT

A methodology is presented for assessing the impact of subgrade variability and related uncertainty on a geotextile-reinforced system’s predicted performance and reliability. The method of analysis is described and results of a sensitivity study are presented with emphasis on how uncertainty on the soft subgrade compressibility affects the reliability of the reinforced soil system.

INTRODUCTION

When roadways are being constructed on soft subgrade, base reinforcement using geosynthetics is an effective alternative to traditional techniques such as chemical stabilization or excavation and substitution. Methods of analysis have been proposed for predicting the mechanical response of two-layer systems reinforced at the interface and have provided rational basis for designing reinforced unpaved roads (e.g. Bender and Barenberg, 1978, Giroud and Noiray, 1981, Gourc, 1982, Sellmeyer et al., 1982, Bonaparte et al., 1984, Love et al., 1987, Milligan et al., 1989, Bourdeau, 1989). A common characteristic to these mechanistic solutions is the underlying assumption of perfectly known subgrade conditions, in spite of the inherent variability of these conditions in practice. The question examined in the present paper is the impact of subgrade variability and related uncertainty on a geotextile-reinforced system’s predicted performance and reliability. The approach taken consists in considering the controlling subgrade property as a random variable and analyzing how the resulting uncertainty propagates through a readily available mechanistic model (Bourdeau, 1989) of the reinforced soil system. The outcome is a probabilistic assessment of the two-layer reinforced system performance where two modes of distress, geotextile tensile failure and insufficient anchorage, are considered.

METHOD OF ANALYSIS

The mechanistic model used in the present study for simulating a two-layer reinforced system is the two-dimensional analysis of tensile membrane action originally proposed by Bourdeau et al. (1982). It is based on the concept that, as the layered system deflects under the load, tension develops in the reinforcement and relieves the pressure on the subgrade (Fig. 1). A detailed formulation can be found elsewhere (Bourdeau, 1989) and only an outline is given herein. The granular base course is represented as a cohesionless particulate medium while the compressible subgrade is represented by a Winkler model.
(Fig. 2). Interface conditions between the geosynthetic and adjacent soil layers follow the Mohr-Coulomb frictional criterion. Stress diffusion theory (Sergeev, 1969, Harr, 1977, Bourdeau and Harr, 1989) is used to compute surface applied load propagation through the granular layer. Then, equilibrium equations of the deflected reinforcing fabric at the interface are solved by means of a finite difference iterative algorithm. Forces acting on the deflected geotextile are shown in Fig. 3. The two conditions for the geotextile to sustain these forces are that the maximal tension does not exceed the tensile strength and sufficient pullout resistance is available through interface friction along the free anchorage length. It is noted that effective reinforcing effect is mobilized according to this mechanism only when deflections are large enough. Within the range of smaller deflections, enhanced confinement of the granular base material would be the predominant reinforcing mechanism.

![Fig. 1. Principle of unpaved road reinforcement through tensile membrane action](image)

(a) Without reinforcement  
(b) With reinforcement

Input parameters to the model are: load strip width, B, and applied pressure, p; granular base course thickness, \( h_1 \), unit weight, \( \gamma \), and stress diffusivity coefficient, \( \nu_1 \); geosynthetic tensile modulus, \( E_m \) and interface friction coefficients, \( F_1 \) and \( F_2 \); subgrade reaction coefficient, \( K_S \). Computed output includes the stress distributions at the base course/ geosynthetic interface and geosynthetic/ subgrade interface, the geosynthetic developed tension, deflection and required anchorage length. The improvement can be measured by the relative reduction of the peak vertical stress on the subgrade as compared to the situation with no reinforcement, as \( (1-k) = [1-(\sigma_{2\text{max}}/\sigma_{1\text{max}})] \). Data and results of a case example computation are shown in Figures 4 and 5.
Fig. 2. Model components

Fig. 3. Forces and stresses on deflected segment of reinforcement

$T$: tensile force in geosynthetic; $\sigma_{1z}$: normal stress on geosynthetic transferred from granular base; $\sigma_{2z}$: normal stress on geosynthetic reacted by subgrade; $\tau_1$: tangential stress on geosynthetic at interface with granular base; $\tau_2$: tangential stress on geosynthetic at interface with subgrade.
Fig. 4. Example of model output: distribution of stresses and tensile force along reinforcement.
Case 1: frictionless interface with edges of reinforcement being fixed;
Case 2: frictional interface with reinforcement edges being free.

Influence of Ks

Fig. 5. Example of model output for Case 2 (free edges and frictional interface): influence of coefficient of subgrade reaction $K_s$ on the maximal tension in reinforcement $T_m$, required anchorage length $L_a$, and improvement ratio $(1-k)$. 
For the probabilistic analysis, the coefficient of subgrade reaction, $K_S$, is considered as a random variable and its first two statistical moments, mean and variance (or its square root, the standard deviation), are used as input parameters. Rosenblueth’s (1975) point estimates method (PEM) is used for computing the resulting first two moments of the maximal tensile force $T_m$, required anchorage length $L_a$ and improvement ratio $(1-k)$ considered as functions of the random variable $K_S$. In the present case where no closed form solution is available and a numerical algorithm must be used for solving the mechanistic model equations, the PEM has the advantage or requiring only a small number of discrete value computations. Furthermore, the method does not require detailed modeling of the random variable probability distribution function. Instead, only a discretized form, based on low order statistical moments, is needed. Details on the PEM procedure can be found elsewhere (e.g. Harr, 1987).

**INFLUENCE OF SUBGRADE COMPRESSIBILITY VARIABILITY ON MODEL RESPONSE**

In order to test how the uncertainty on the coefficient of subgrade reaction affects the system’s response, sensitivity study was performed, base on the example used in the previous section (i.e. Case 2 with free geotextile edges and frictional interface). The same geometry and data shown in Fig. 4 were used for the parameters considered deterministic (i.e. all but $K_S$) and were kept constant. The value of 2000 KN/m$^3$ was also adopted for the mean value of the random variable $K_S$ and kept constant, while its standard deviation was varied between 0 and 1500 kN/m$^3$. In relative value, this represents for $K_S$ a coefficient of variation $V_{K_S}$ (i.e. the ratio of the standard deviation to the mean) between 0% and 75%. The coefficient of subgrade reaction is correlated, for instance, to the CBR coefficient or to plate load tests results and, similarly to these test results, large values of its coefficient of variation can be anticipated for soft subgrades. Resulting mean values of $T_m$, $L_a$ and $(1-k)$ are plotted in function of $V_{K_S}$ in Fig. 6. All three curves reveal the expected values of the model responses vary at non-linear rates under the influence of the subgrade variability. This information has to be considered together with the plots of Fig. 7 where the influence of $V_{K_S}$ on the coefficients of variation of the model outputs is shown. The improvement provided by the geotextile reinforcement $(1-k)$ is, on average, more effective when there is more variability in subgrade conditions but it is also subject to larger uncertainty. Longer anchorage length is required, on average, when $V_{K_S}$ increases, and the standard deviation of $L_a$ increases very rapidly at the same time. The maximal tensile force is also affected, but to a lesser extent than the other two characteristics of the system’s response.

Further analysis can be carried out if the probability distributions of the system response functions are modeled consistently with the PEM results. In the present case, the mean values and standard deviations are computed, and the physical meaning of $T_m$, $L_a$ and $(1-k)$ forbids negative values. Lognormal distributions are consistent with these constraints and can be fitted accordingly. A conceptual representation of the distribution is shown for the required anchorage length $L_a$ in Fig. 8 where the dashed area beyond a selected design length $L$ represents the probability that this design length is insufficient. The
dashed area represents the probability of anchorage “failure” and the complement area below the probability density curve represents the anchorage reliability. Similarly, the distribution of the maximal tensile force can be modeled in order to compute the probability of tensile failure. Using this information, it is possible to determine what safety factors would be required for an accepted probability of failure (or a target reliability). In the present context, safety factors are defined with respect to anchorage and tensile resistance, respectively as $F_{SL}$, the ratio of the design length $L$ to the mean value of required length $L_{a}$, and $F_{ST}$, the ratio of the geotextile tensile strength to the mean value of the peak tensile force $T_{m}$. Required safety factors were computed for a probability of failure of 5% relative to each mode of distress, over the range of coefficient of variation for $K_{S}$. Results are shown in Fig. 9. These confirm the dominance of the anchorage mode of distress over the tensile failure mode. For instance, with $V_{K_{S}} = 30\%$, a safety factor of 1.2 on the tensile strength would provide a 95% reliability while a safety factor of 2.5 on the anchorage length would be needed in order to satisfy the same criterion.

Fig. 6. Influence of coefficient of variation of $K_{S}$ on mean values of $(1-k)$, $T_{m}$ and $L_{a}$
Fig. 7. Influence of coefficient of variation of $K_s$ on coefficients of variation of $(1-k)$, $T_m$ and $L_a$.

Fig. 8. Probability distribution of required anchorage length $L_a$, probability of failure $P_L$ and reliability $R_L$ with respect to anchorage (conceptual representation).
CONCLUDING COMMENTS

The methodology proposed for assessing the impact of subgrade variability on geotextile-reinforced unpaved road performance and reliability is presented herein for a specific mechanistic model of soil-reinforcement interaction. This particular model is selected for the purpose of illustrating the methodology in the case of membrane mechanism with geotextile reinforcement but, thanks to the PEM robustness, other models can be integrated into the same approach for addressing other mechanisms of reinforcement and geosynthetic types. Based on the assumptions and example data used in the present study, the following observations are made:

- Subgrade variability modeled as randomness in the controlling compressibility parameter affects both the mean values and standard deviations of the model response functions (maximal tensile force in reinforcement, required anchorage length and reinforcement effectiveness).
- The dominant mode of distress is the anchorage pullout, as compared to tensile failure.

However, in its present state of development, the method used in this study includes a number of simplifications that will have to be revisited. Among these is the translation of spatial variability of subgrade conditions along the roadway into a lumped random
variable used for analyzing a two-dimensional cross-section. Further investigation is needed if spatial variability is to be modeled more accurately.

REFERENCES


